Midterm Exam
(Math 200 A, Fall 06)

Solve the following problems. Show all your work in the space under each problem.

1. Answer the following: (15 pts)

(a) Evaluate the integral: $\int_1^2 (1+\frac{1}{x})\,dx$

$\int_1^2 (1+\frac{1}{x})\,dx = [x+\ln x]_1^2 = 2+\ln 2 - 1 - \cancel{\ln 1}^{\,0} = 1+\ln 2$ //

(b) The result you found in (a) describes:

A. The <u>length</u> of the arc of the hyperbola $y=1+\frac{1}{x}$ from $x=1$ to $x=2$.

B. The <u>area</u> under the hyperbola $y=1+\frac{1}{x}$ from $x=1$ to $x=2$.

C. The <u>volume</u> by revolution around the x-axis generated by the hyperbola $y=1+\frac{1}{x}$ from $x=1$ to $x=2$.

(c) <u>True</u> or <u>False</u>: $\int_a^a f(x)\,dx = 0$ True //

2. Answer the following: (20 pts)

(a) Evaluate the integral: $\int \frac{1-x}{\sqrt{2x-x^2}}\,dx$ (use substitution method)

Let $2x-x^2=u^2 \Rightarrow (2-2x)dx = 2u\,du \Rightarrow \cancel{2}(1-x)\,dx = \cancel{2}u\,du \Rightarrow (1-x)\,dx = u\,du$

So, $\int \frac{1-x}{\sqrt{2x-x^2}}\,dx = \int \frac{u\,du}{\sqrt{u^2}} = \int \frac{\cancel{u}}{\cancel{u}}\,du = \int du = u = \sqrt{2x-x^2}+C$ //

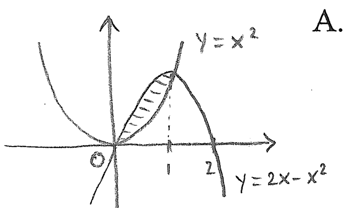
(b) Evaluate the integral: $\int x\cos x\,dx$ (use by parts method)

$\int x\cos x\,dx = \int x\,d(\sin x) = x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$ //

3. (a) The area of the region enclosed by the parabolas $y=2x-x^2$ and $y=x^2$ is: (20 pts)

A. 10/6 B. 2/3 C. 1/3 D. 5/6



They intersect at: $2x-x^2=x^2 \Rightarrow x(1-x)=0 < \begin{matrix} x=0 \\ x=1 \end{matrix}$

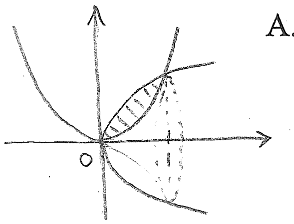
$A=\int_0^1 (y_1-y_2)\,dx = \int_0^1 (2x-x^2-x^2)\,dx$

$= \int_0^1 (2x-2x^2)\,dx = 2\int (x-x^2)\,dx$

$= 2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 1-\frac{2}{3}-0+0 = \frac{1}{3}$ //

(b) The volume of the solid obtained by rotating the region bounded by the curves

$y=\sqrt{x}$ and $y=x^2$ around the x-axis is:



A. 3/10 　 B. $3\pi/10$ 　 C. $3\pi/5$ 　 D. $10\pi/3$

They intersect at : $\sqrt{x}=x^2 \Rightarrow x=x^4 \Rightarrow x(1-x^3)=0 \left\langle \begin{matrix} x=0 \\ x=1 \end{matrix}\right.$

So, $V=\pi\int_0^1 (y_1^2-y_2^2)\,dx = \pi\int_0^1 [(\sqrt{x})^2-(x^2)^2]\,dx = \pi\int_0^1 (x-x^4)\,dx$

$= \pi\left[\frac{x^2}{2}-\frac{x^5}{5}\right]_0^1 = \pi(1/2-1/5) = 3\pi/10$

4. (a) Evaluate the integral: $\int \sin^2 x\cos^3 x\,dx$ (15 pts)

$\int \sin^2 x\cos^3 x\,dx = \int \sin^2 x\cos^2 x\cos x\,dx = \int \sin^2 x(1-\sin^2 x)\cos x\,dx$

$= \int(\sin^2 x-\sin^4 x)\cos x\,dx \underset{du=\cos x dx}{\overset{u=\sin x}{=}} \int(u^2-u^4)\,du = \frac{u^3}{3}-\frac{u^5}{5} = \frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$ //

(b) The correct substitution to evaluate the integral $\int \frac{dx}{x^2\sqrt{x^2-4}}$ is:

A. $x=2\sin\theta$ 　 B. $x=2\tan\theta$ 　 C. $x=2\cos\theta$ 　 D. $x=2\sec\theta$

It is D bec. it leads to a simplified computable integral.

ie, $dx=2\sec\theta\tan\theta\,d\theta$, so $\int \frac{2\sec\theta\tan\theta}{4\sec^2\theta\sqrt{4\sec^2\theta-4}}\,d\theta = \int\frac{\cancel{2}\sec\theta\cancel{\tan\theta}}{4\sec^2\theta\,\cancel{2\tan\theta}}\,d\theta = \frac{1}{4}\int\frac{1}{\sec\theta}\,d\theta = \frac{1}{4}\int\cos\theta\,d\theta = \frac{1}{4}\sin\theta = \ldots$

(c) The correct partial fraction expansion to evaluate the integral $\int\frac{10}{(x-1)(x^2+9)}\,dx$ is:

A. $\frac{A}{x-1}+\frac{B}{x^2+9}$ 　 B. $\frac{A}{x-1}+\frac{Bx+C}{x^2+9}$ 　 C. $\frac{A}{x-1}+\frac{Bx}{x^2+9}$ 　 D. $\frac{A}{x-1}+\frac{B}{(x^2+9)^2}$

5. Evaluate the improper integral: $\int_{-\infty}^0 xe^x\,dx$ (10 pts)

$\int_{-\infty}^0 xe^x\,dx = \lim_{t\to-\infty}\int_t^0 xe^x\,dx \overset{\text{By Parts}}{=} \lim_{t\to-\infty}\int_t^0 x\,d(e^x) = \lim_{t\to-\infty}\left([xe^x]_t^0-\int_t^0 e^x\,dx\right)$

$= \lim_{t\to-\infty}\left(0-te^t-[e^x]_t^0\right) = \lim_{t\to-\infty}[-te^t-1+e^t] = \lim_{t\to-\infty}\left[-\frac{t}{e^{-t}}-1+e^t\right]$

$= \frac{\infty}{\infty} \overset{LH}{=} \lim_{t\to-\infty}[0-1+0] = -1$

6. The length of the arc of the curve $x=2y^{3/2}$ from (0,0) to (2,1). (10 pts)

$L=\int_0^1\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = \int_0^1\sqrt{1+(3y^{1/2})^2}\,dy = \int_0^1\sqrt{1+9y}\,dy = \int_0^1(1+9y)^{1/2}\,dy$

$= \frac{1}{9}\cdot\frac{2}{3}\left[(1+9y)^{3/2}\right]_0^1 = \frac{2}{7}(10^{3/2}-1)$ //

7. Given the curve $x=t-t^2$, $y=t-t^3$, with $-1<t<1$, find $\frac{d^2y}{dx^2}|_{(0,0)}$. (10 pts)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-3t^2}{1-2t}$

$\frac{d^2y}{dx^2} = \frac{d/dt\,(dy/dx)}{dx/dt} = \frac{\frac{(1-2t)(-6t)-(1-3t^2)(-2)}{(1-2t)^2}}{1-2t} = \frac{6t^2-6t+2}{(1-2t)^3}$

At the point (0,0) we have $\begin{cases} 0=t-t^2 \Rightarrow 0=t(1-t) \Rightarrow t=0,\ \cancel{t=1} \\ 0=t-t^3 \Rightarrow 0=t(1-t^2) \Rightarrow t=0,\ \cancel{t=\pm1}\end{cases}$, ie $t=0$

ie, $\frac{d^2y}{dx^2}\Big|_{t=0} = \frac{0-0+2}{(1-0)^3} = 2$ //